Mathematical Modeling of Weather-Induced Degradation of Polymer Properties

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SYNOPSIS

Weather-induced degradation of polymer properties is caused by all the factors of weather, which include solar radiation, temperature, humidity, wind, rain, environmental pollutants, thermal cycling (cold night and hot days), and sand abrasion. Linear low-density polyethylene (LLDPE) is exposed to natural weather, and degradation is monitored by the mechanical properties testing system, Fourier transform infrared (FTIR) spectroscopy, and differential scanning calorimetry (DSC). Three mathematical models were developed with weather parameters as independent parameters and mechanical property (tensile strength), chemical change (carbonyl growth), and thermal property (percent crystallinity) as dependent parameters. The mechanical property was found to be more dependent on the ultraviolet (UV) portion of the total solar radiation, chemical change was found to be synergestically effected by UV and total solar radiation, and change in thermal property was because of UV, total solar radiation, and temperature. Humidity and other weather parameters were found to play a less significant role in the weather-induced degradation of LLDPE properties.

INTRODUCTION

Models are representative of objects, processes, or systems that are to be described or whose patterns of behavior are to be analyzed. These models are mathematical if the representations are mathematical relationships. The mathematical model solution in many cases requires a computational/simulation approach. It is now widely acknowledged that, along with the traditional and theoretical methodologies, advanced work in various areas of science and engineering has come to rely critically on the computational/simulation approach.

The weathering of plastics is dependent on almost all parameters of environment. The weather is so variable from time to time and from place to place that even comparison among outdoor tests obtained at different seasons, years, or locations have been inadequate. A mathematical approach in describing the weather-induced degradation of plastics is considered for the purpose of experimental data presentation, prediction, and understanding of this complex phenomena.[2] The previous publications on this subject have already demonstrated that the weathering of plastic is inherently related to weather variables.[3-6]

Regression analysis is a statistical technique for modeling and investigating the relationship between the dependent and independent variables. Its broad appeal results from the conceptually simple process of using an equation to express the relationship between a set of variables. In the field of plastic weathering, regression analysis can be used to build a model that expresses degradation in significant properties of plastic as a function of weather parameters.

In this work, statistical techniques will be used to determine the significant weather parameters influencing the decay in important properties of plastic. Based on these parameters, three different mathematical models will be developed representing the degradation in mechanical property (tensile strength), chemical structure (carbonyl groups), and thermal property (percent crystallinity). Selection of weather parameters significant for a specific model will be accomplished using the stepwise

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regression analysis technique.[7] Finally, mathematical models will be developed using multiple linear regression and residual analysis will also be presented to evaluate the goodness of fit. In the present analysis, computation has been carried out using a statistical analysis system (SAS) software package on mainframe IBM 3033 computer.

EXPERIMENTAL

Degradation of plastics during outdoor exposure is influenced to varying degrees by all natural climatic phenomena. Heat, radiation (UV and IR), rain, humidity, and atmospheric contaminants all contribute to the degradation of plastics subjected to outdoor exposure. None of these phenomena is constant in one location, and weather conditions vary widely with location. To attain maximum accuracy in predicting the useful life of an outdoor plastic, all aspects of the anticipated environment to which it will be exposed should be considered. This is best accomplished by conducting actual outdoor exposure

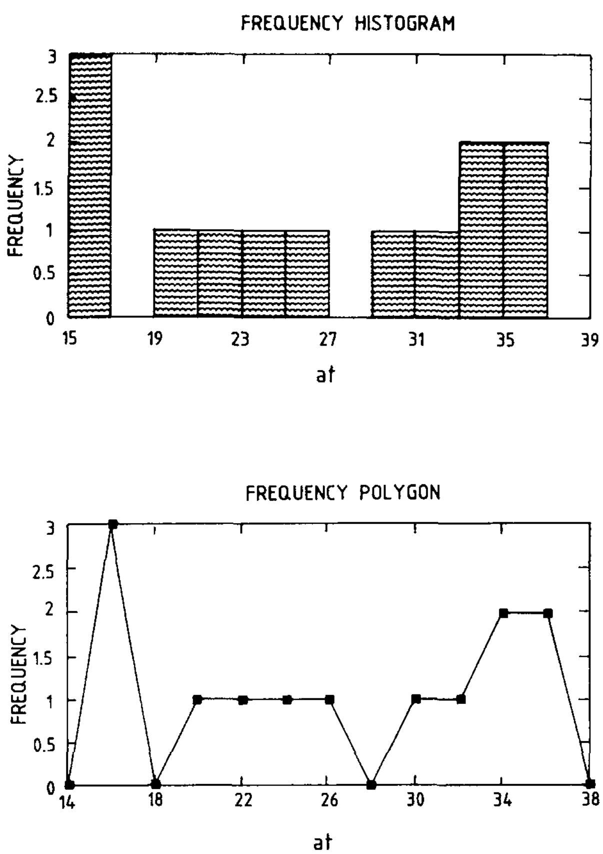

Figure 1 Frequency histogram and polygon of average monthly temperature (at).

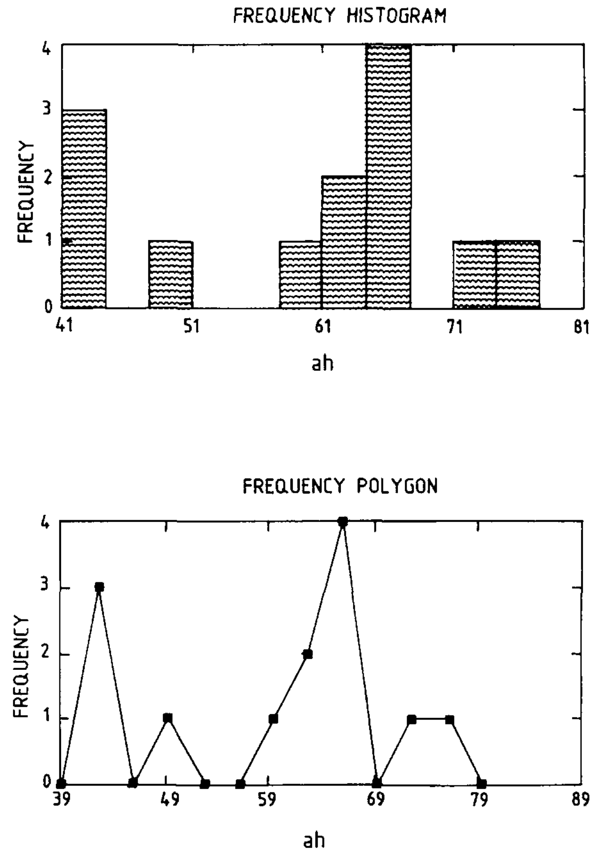

Figure 2 Frequency histogram and polygon of average monthly humidity (ah).

trials.[8] In this work, LLDPE was selected for studying the effects of the severe natural weather of Dhahran, Saudi Arabia, on its significant properties. Weathering trials in hot climatic regions (such as Saudi Arabia) are of particular importance. Almost invariably, the high levels of temperature, humidity, and solar radiation found in such regions prove more aggressive to plastic materials than do the conditions in cold regions (such as England). Thus, as well as being of intrinsic interest, tropical and subtropical exposure trials are a means of providing accelerated exposure sites as compared to colder regions like England.

In order to assess the durability of polymer, it is mandatory to expose it to natural weather or to simulated conditions of UV radiation, temperature, rain, and humidity. It is unlikely that any one meteorological element is the sole contributor to the degradation of plastics exposed to outdoor conditions. The complete phenomenon of weathering involves the combined effects of photo and thermally initiated oxidation and ozonolysis, associated with the purely physical effects of wind, temperature variation, and

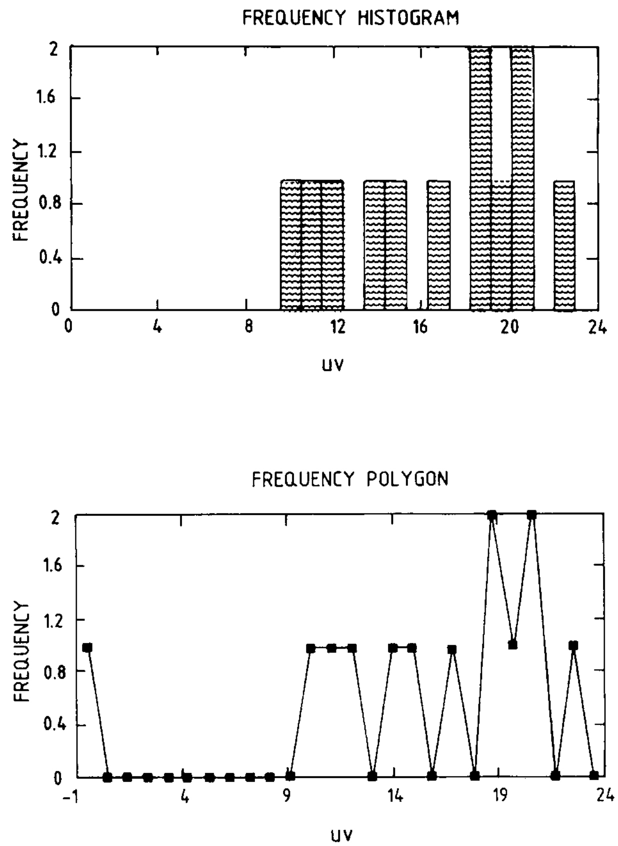

Figure 3 Frequency histogram and polygon of average monthly UV radiation (uv).

humidity variation. These work together in the breakdown of the material.

A comparison of the levels of total solar radiation received in various parts of world revealed that Saudi Arabia receives a high dose of total solar radiation.[8] In Saudi Arabia, the heavy dose of solar radiation and temperature reaching up to 50°C in summer and severe thermal cycling would result in extreme thermal stresses in the specimen. Such a combination of very high UV dose, temperature extremes, and thermal cycling proves to be extremely aggressive to the plastic and results in a much faster rate of degradation of plastic than is observed in other parts of the world. Dhahran's weather could be considered as a naturally accelerated laboratory to evaluate the weathering resistance of plastics.

Materials

The polymer used in the study was linear low-density polyethylene (LLDPE) in pellet form containing no UV stabilizer and identified as Ladene FH10018 (Saudi Basic Industries Corporation [SABIC]). The polymer grade used is a slip, antiblock, and antioxidant-modified LLDPE resin (SABIC Marketing, 1984).

The test sheets were compression-molded using a Carver laboratory press for films and Wabash 75 tons press for plaques in accordance with the American Society for Testing Standard Material (ASTM) standard (ASTM Standards D-1928, 1980). The press is provided with platens that can be heated to 200°C using electrical resistance heaters. It is designed so as to provide maximum heat without the occurrence of "hot spots" and maintains the rigidity of the plates. Cooling was accomplished by passing water through channels provided for this purpose. The chases used were single-cavity picture frame molds with dimensions appropriate to the production of test sheets, 140 micron 6 × 6 in. films and 1/16 × 16 × 16 in. plaques. Flat backing plates for the chases were strong enough to resist warping or distortion by molding. Stainless steel plates of the same length and width as the outside chase dimension were employed. Aluminum foil 0.05 mm thick

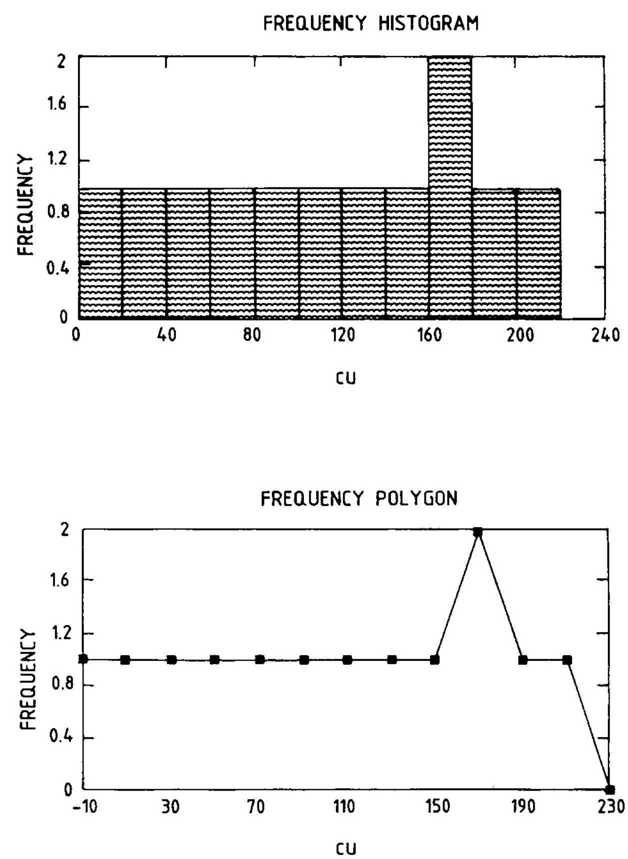

Figure 4 Frequency histogram and polygon of cumulative UV radiation (cu).

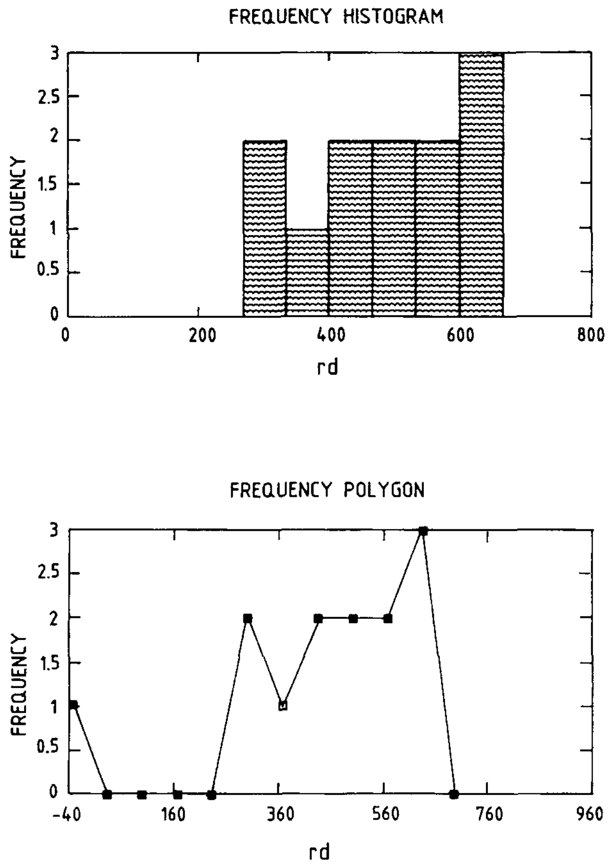

Figure 5 Frequency histogram and polygon of average monthly total solar radiation (rd).

was used as a parting agent in the molding operation. Test specimens were prepared from the test sheets using blanking die but without disturbing the thermal history introduced during sheet preparation, which provided specimens of an acceptable quality, as judged by visual examination.

Meteorological and Radiation Environment of Test Site

Dhahran (26.32°N, 50.13°E) is situated just north of the Tropic of Cancer on the eastern coastal plain of Saudi Arabia and is close to 10 km inland from the Arabian Gulf. Despite its nearness to the coast, Dhahran is located in very much a desert environment. The environment of the site plus the limited human activities and population mean that the radiation characteristics of the atmosphere are not significantly altered by manmade pollution sources.

Four distinct seasons cannot be identified in the classical midlatitude sense. Rather, the year may be divided into a very hot period and a cooler period. For the Dhahran region, this division may be set at the maximum change between monthly mean temperature, giving the separation into the two 6-month intervals: May to October (hot) and November to April (cooler).

Annual precipitation totals are very low, typically around 80 mm in Dhahran and somewhat less inland; 60% falls in December/January, and there is no rain at all from June to October during most of the years. Wind speed show a clear diurnal variability within the typical range from near zero to 10 m/s; there is no regular diurnal march. The synoptic wind direction exhibits a long period of more or less constant direction between north and northwest, though this synoptic flow is overlaid with a sea/land breeze. An additional feature with some longevity is the tendency for the wind to swing to the east, in particular, to the quadrant between east and south.

The parameter of most general interest in Saudi Arabia is always the temperature. At Dhahran, monthly mean temperatures reach close to 37°C for both July and August, with daily maxima often ap-

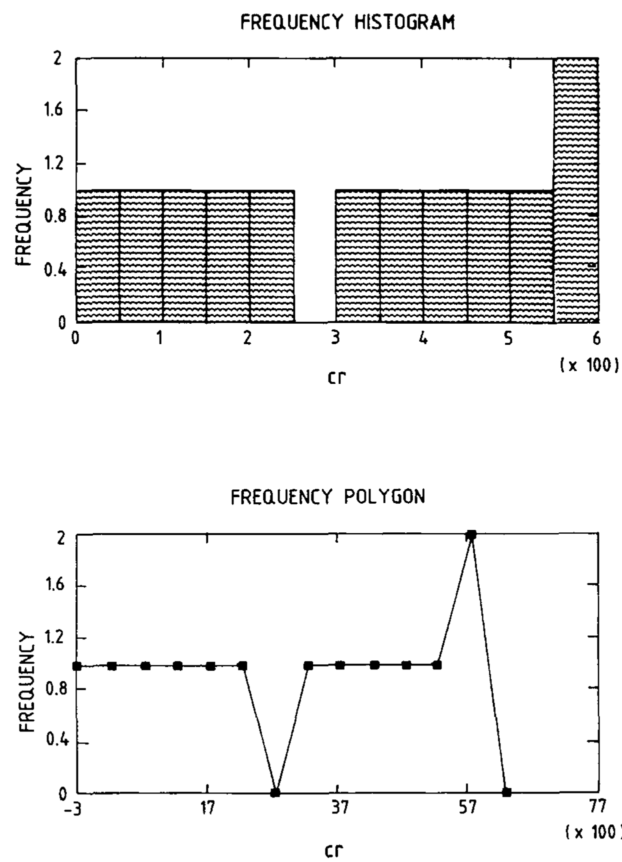

Figure 6 Frequency histogram and polygon of cumulative total solar radiation (cr).

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FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE TS

STEP 1     VARIABLE CU ENTERED          R SQUARE = 0.94188759
                                        C(P) =    236.76583913
                DF     SUM OF SQUARES  MEAN SQUARE        F    PROB>F
REGRESSION       1     51459.37660039  51459.37660   178.29    0.0001
ERROR           11      3174.93109192    288.63010
TOTAL           12     54634.30769231
                B VALUE     STD ERROR   TYPE II SS        F    PROB>F
INTERCEPT  180.2620773
CU          -0.9187786    0.06880966   51459.37660   178.29    0.0001
BOUNDS ON CONDITION NUMBER:             1,             1
------------------------------------------------------------------
STEP 2     VARIABLE UV ENTERED          R SQUARE = 0.99635160
                                        C(P) =      8.42960718
                DF     SUM OF SQUARES  MEAN SQUARE        F    PROB>F
REGRESSION       2     54434.97999182  27217.49000  1365.46    0.0001
ERROR           10       199.32770049     19.93277
TOTAL           12     54634.30769231
                B VALUE     STD ERROR   TYPE II SS        F    PROB>F
INTERCEPT  214.5621089
UV          -2.6135216    0.21390572    2975.60339   149.28    0.0001
CU          -0.8574241    0.01876697   41607.39820  2087.39    0.0001
BOUNDS ON CONDITION NUMBER:      1.077119,      4.308475
------------------------------------------------------------------
STEP 3     VARIABLE AT ENTERED          R SQUARE = 0.99843059
                                        C(P) =      1.63727803
                DF     SUM OF SQUARES  MEAN SQUARE        F    PROB>F
REGRESSION       3     54548.56387834  18182.85463  1908.54    0.0001
ERROR            9        85.74381397      9.52709
TOTAL           12     54634.30769231
                B VALUE     STD ERROR   TYPE II SS        F    PROB>F
INTERCEPT  220.5149542
AT          -0.5800969    0.16800505     113.58389    11.92    0.0072
UV          -2.1178201    0.20610607    1005.90465   105.58    0.0001
CU          -0.8425488    0.01367105   36186.53117  3798.28    0.0001
BOUNDS ON CONDITION NUMBER:      2.319667,      16.82329
------------------------------------------------------------------
NO OTHER VARIABLES MET THE 0.0500 SIGNIFICANCE LEVEL FOR ENTRY
SUMMARY OF FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE TS
      VARIABLE  NUMBER     PARTIAL      MODEL
STEP  ENTERED       IN        R**2       R**2         C(P)
   1  CU             1      0.9419     0.9419      236.766
   2  UV             2      0.0545     0.9964        8.430
   3  AT             3      0.0021     0.9984        1.637
      VARIABLE
STEP  ENTERED            F      PROB>F   LABEL
   1  CU          178.2883      0.0001   CUMULATIVE UV RADIATION
   2  UV          149.2820      0.0001   UV RADIATION
   3  AT           11.9222      0.0072   MONTHLY AVERAGE TEMPERATURE
------------------------------------------------------------------
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Figure 7 Computer output (SAS) of stepwise forward selection procedure applied to LLDPE tensile strength data.

proaching the 50°C mark. However, the eastern coastal climatic region of Saudi Arabia is a region where significant year-end cooling is in evidence and monthly mean temperatures in the cooler season are some 20°C lower than in the hottest months. Despite the desert location, the nearness of the very shallow Arabian Gulf (average depth 30 m) means that relative humidity values are relatively high. The

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BACKWARD ELIMINATION PROCEDURE FOR DEPENDENT VARIABLE TS

STEP 0     ALL VARIABLES ENTERED         R SQUARE = 0.99858127
                                         C(P) =     7.00000000
                 DF    SUM OF SQUARES    MEAN SQUARE            F     PROB>F
REGRESSION        6    54556.79656782    9092.7994280      703.86     0.0001
ERROR             6       77.51112449      12.9185207
TOTAL            12    54634.30769231
                 B VALUE     STD ERROR    TYPE II SS            F     PROB>F
INTERCEPT    223.71602893
AT            -0.66596961   0.29662351   65.11941108         5.04     0.0659
AH            -0.03576751   0.21996207    0.34158168         0.03     0.8762
RD             0.06228173   0.14520012    2.37684146         0.18     0.6829
UV            -3.86652610   4.24373749   10.72402102         0.83     0.3974
CR            -0.00306899   0.19224686    0.00329220         0.00     0.9878
CU            -0.75164105   5.57965056    0.23443355         0.02     0.8972
BOUNDS ON CONDITION NUMBER:      147326.3,        1773238
-----------------------------------------------------------------------------
STEP 1     VARIABLE CR REMOVED           R SQUARE = 0.99858121
                                         C(P) =     5.00025484
                 DF    SUM OF SQUARES    MEAN SQUARE            F     PROB>F
REGRESSION        5    54556.79327562   10911.358655       985.36     0.0001
ERROR             7       77.51441669     11.073488
TOTAL            12    54634.30769231
                 B VALUE     STD ERROR    TYPE II SS            F     PROB>F
INTERCEPT    223.81007022
AT            -0.66446342   0.26036275     72.122188         6.51     0.0380
AH            -0.03818386   0.14776395      0.739447         0.07     0.8035
RD             0.06036701   0.07576728      7.029435         0.63     0.4518
UV            -3.80979869   2.14785921     34.839898         3.15     0.1194
CU            -0.84071275   0.02210916  16011.618821      1445.94     0.0001
BOUNDS ON CONDITION NUMBER:      203.4466,       2047.439
-----------------------------------------------------------------------------
STEP 2     VARIABLE AH REMOVED           R SQUARE = 0.99856768
                                         C(P) =     3.05749415
                 DF    SUM OF SQUARES    MEAN SQUARE            F     PROB>F
REGRESSION        4    54556.05382844   13639.013457      1394.34     0.0001
ERROR             8       78.25386387      9.781733
TOTAL            12    54634.30769231
                 B VALUE     STD ERROR    TYPE II SS            F     PROB>F
INTERCEPT    220.82661930
AT            -0.61758948   0.17554466    121.071055        12.38     0.0079
RD             0.06207538   0.07093946      7.489950         0.77     0.4071
UV            -3.86575104   2.00841490     36.239041         3.70     0.0904
CU            -0.84490637   0.01411211  35062.998573      3584.54     0.0001
BOUNDS ON CONDITION NUMBER:      201.8977,       1596.415
-----------------------------------------------------------------------------
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Figure 8 Computer output (SAS) of stepwise backward elimination procedure applied to LLDPE tensile strength data.

relative humidity exhibits a large diurnal cycle on the order of 60% throughout the year, with daily maxima often rising over the 80% level during most months.

The desert location, the prevailing wind direction, and the relatively strong winds often experienced all combine to mean that the lower atmosphere almost always possesses a significant dust/sand content. A detailed assessment of the atmospheric turbidity has been undertaken.[9]

Natural Exposure

The outdoor weathering of plastics can be used to evaluate the stability of plastic materials that are

```
STEP 3    VARIABLE RD REMOVED          R SQUARE = 0.99843059
                                       C(P) =      1.63727803
               DF     SUM OF SQUARES   MEAN SQUARE        F      PROB>F
REGRESSION      3     54548.56387834   18182.854626   1908.54    0.0001
ERROR           9        85.74381397       9.527090
TOTAL          12     54634.30769231
                B VALUE    STD ERROR    TYPE II SS         F      PROB>F
INTERCEPT  220.51495418
AT          -0.58009691   0.16800505    113.583887     11.92    0.0072
UV          -2.11782006   0.20610607   1005.904655    105.58    0.0001
CU          -0.84254882   0.01367105  36186.531173   3798.28    0.0001
BOUNDS ON CONDITION NUMBER:      2.319667,      16.82329
---------------------------------------------------------------------------
ALL VARIABLES IN THE MODEL ARE SIGNIFICANT AT THE 0.0500 LEVEL.
SUMMARY OF BACKWARD ELIMINATION PROCEDURE FOR DEPENDENT VARIABLE TS
         VARIABLE   NUMBER    PARTIAL    MODEL
STEP     REMOVED       IN       R**2      R**2        C(P)
   1     CR             5     0.0000    0.9986     5.00025
   2     AH             4     0.0000    0.9986     3.05749
   3     RD             3     0.0001    0.9984     1.63728
         VARIABLE
STEP     REMOVED              F        PROB>F     LABEL
   1     CR              0.0003        0.9878     CUMULATIVE RADIATION
   2     AH              0.0668        0.8035     MONTHLY AVERAGE HUMIDITY
   3     RD              0.7657        0.4071     TOTAL SOLAR RADIATION
---------------------------------------------------------------------------
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Figure 8 *(continued from the previous page)*

exposed to varied meteorological influences. In this study, the outdoor weathering of LLDPE was carried out according to ASTM Standard (1979), and British (1981) standards on exposure to natural weathering were also taken into consideration.

The racks were placed in such a location that no shadow from a neighboring obstruction with an angle of elevation greater than 20° fell on any sample. The racks were adjusted so that the exposed surfaces of the samples were at an angle of 45° to the horizontal and facing south.[10] Racks were constructed of untreated wood, which is recommended for desert areas (ASTM Standard D-1435, 1979).

The samples for exposure testing were mounted on holders, and the evaluation samples were cut in such a way that the mounting edges were removed in cases where the test results might otherwise be affected. The effect of backing was considered important in these weathering trials, and the rack was so designed to expose the samples from both sides. Backing contributes to the degradation process with regard to reflectance, heat absorption, etc. The total number of samples was 60, and withdrawal frequency was maintained on a monthly basis for a total exposure of 1 year (1986). Five samples were withdrawn at each interval, and one sample was exposed for the complete 12 months except when withdrawn for FTIR analysis. Similarly, five samples were withdrawn each for thermal analysis (DSC) and mechanical testing.

Since one can study exactly the same portion of the sample, spectral subtractions are made on a one-to-one basis during the early stages of the reaction. The resultant difference in spectra can be magnified to bring out small spectral features. The control samples were retained for determination of original and final control values. The control and withdrawn samples were retained at standard conditions of 23 ± 1°C and 50 ± 2% relative humidity. They were covered with inert wrapping to prevent light exposure during the aging period.

MODELING

Variable Description

The independent variables considered are the significant weather parameters. Mathematically,

$$\text{Degradation (DG)} = F\,(AT, AH, UV, CU, RD, CR) \quad (1)$$

where DG = degradation of significant plastic property, AT = average monthly temperature (°C), AH

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STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE TS

STEP 1    VARIABLE CU ENTERED           R SQUARE = 0.94188759
                                        C(P) =    236.76583913
                 DF   SUM OF SQUARES   MEAN SQUARE          F    PROB>F
REGRESSION        1  51459.37660039   51459.37660    178.29    0.0001
ERROR            11   3174.93109192     288.63010
TOTAL            12  54634.30769231
               B VALUE    STD ERROR     TYPE II SS          F    PROB>F
INTERCEPT   180.262077
CU           -0.918779   0.06880966   51459.37660    178.29    0.0001
BOUNDS ON CONDITION NUMBER:                1,              1
-------------------------------------------------------------------------
STEP 2    VARIABLE UV ENTERED           R SQUARE = 0.99635160
                                        C(P) =      8.42960718
                 DF   SUM OF SQUARES   MEAN SQUARE          F    PROB>F
REGRESSION        2  54434.97999182   27217.49000   1365.46    0.0001
ERROR            10    199.32770049      19.93277
TOTAL            12  54634.30769231
               B VALUE    STD ERROR     TYPE II SS          F    PROB>F
INTERCEPT   214.562109
UV           -2.613522   0.21390572    2975.60339    149.28    0.0001
CU           -0.857424   0.01876697   41607.39820   2087.39    0.0001
BOUNDS ON CONDITION NUMBER:      1.077119,      4.308475
-------------------------------------------------------------------------
STEP 3    VARIABLE AT ENTERED           R SQUARE = 0.99843059
                                        C(P) =      1.63727803
                 DF   SUM OF SQUARES   MEAN SQUARE          F    PROB>F
REGRESSION        3  54548.56387834   18182.85463   1908.54    0.0001
ERROR             9     85.74381397       9.52709
TOTAL            12  54634.30769231
               B VALUE    STD ERROR     TYPE II SS          F    PROB>F
INTERCEPT   220.514954
AT           -0.580097   0.16800505     113.58389     11.92    0.0072
UV           -2.117820   0.20610607    1005.90465    105.58    0.0001
CU           -0.842549   0.01367105   36186.53117   3798.28    0.0001

BOUNDS ON CONDITION NUMBER:      2.319667,      16.82329
-------------------------------------------------------------------------
NO OTHER VARIABLES MET THE 0.0500 SIGNIFICANCE LEVEL FOR ENTRY
SUMMARY OF
STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE TS
           VARIABLE           NUMBER    PARTIAL     MODEL
STEP  ENTERED   REMOVED          IN       R**2       R**2        C(P)
   1  CU                          1     0.9419     0.9419     236.766
   2  UV                          2     0.0545     0.9964       8.430
   3  AT                          3     0.0021     0.9984       1.637
           VARIABLE
STEP  ENTERED   REMOVED             F                PROB>F
   1  CU                      178.2883               0.0001
   2  UV                      149.2820               0.0001
   3  AT                       11.9222               0.0072
-------------------------------------------------------------------------
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Figure 9 Computer output (SAS) of stepwise regression procedure applied to LLDPE tensile strength data.

= average monthly relative humidity (%), UV = average monthly UV radiation dose (Langleys), CU = cumulative monthly UV radiation (Langleys), RD = average total solar radiation (Langleys), and CR = cumulative total solar radiation (Langleys).

The descriptive statistical analysis of weather

N=13 REGRESSION MODELS FOR DEPENDENT VARIABLE: TS
MODEL: MODEL1

NUMBER IN MODEL	R-SQUARE	C(P)	VARIABLES IN MODEL
1	0.04273724	4039.404	AH
1	0.23478987	3227.185	UV
1	0.25087678	3159.151	RD
1	0.32497833	2845.765	AT
1	0.93928701	247.764	CR
1	0.94188759	236.766	CU
2	0.28997257	2995.810	UV RD
2	0.33608978	2800.773	AT UV
2	0.33939210	2786.807	AT RD
2	0.39532613	2550.254	UV AH
2	0.41834753	2452.893	AH RD
2	0.69210519	1295.132	AT AH
2	0.96022382	161.219	AH CR
2	0.96137511	156.350	CU AH
2	0.97145251	113.731	CU CR
2	0.97956231	79.433970	AT CR
2	0.98001899	77.502590	AT CU
2	0.99561044	11.564077	RD CR
2	0.99603117	9.784754	CU RD
2	0.99603223	9.780297	UV CR
2	0.99635160	8.429607	CU UV
3	0.35679148	2715.223	AT UV RD
3	0.46363196	2263.379	UV AH RD
3	0.69647921	1278.634	AT UV AH
3	0.69785458	1272.817	AT AH RD
3	0.97199261	113.447	CU AH CR
3	0.98002991	79.456421	AT AH CR
3	0.98054565	77.275273	AT CU AH
3	0.98215082	70.486789	AT CU CR
3	0.99603963	11.748981	UV RD CR
3	0.99635165	10.429392	CU UV RD
3	0.99646937	9.931535	AH RD CR
3	0.99671527	8.891603	CU AH RD
3	0.99708236	7.339136	UV AH CR
3	0.99711775	7.189460	CU UV CR
3	0.99720840	6.806094	CU UV AH
3	0.99734192	6.241402	CU RD CR
3	0.99772904	4.604229	AT RD CR
3	0.99790438	3.862695	AT CU RD
3	0.99835219	1.968823	AT UV CR
3	0.99843059	1.637278	AT CU UV

Figure 10 Computer output (SAS) of RSQUARE and Mallow's Cp procedure applied to LLDPE tensile strength data.

data was carried out with the purpose of viewing the frequency distribution and determining some potential outliers that can misinterpret the total behavior.[11] Frequency distribution and histograms of weather parameters are presented in Figures 1–6 for AT, AH, UV, CU, RD, and CR, respectively.

Variables Selection

The well-established theoretical background of weather-induced degradation of polyethylene indicate that the regressor variables (weather parameters) included are influential. Some of the weather

4	0.70551227	1242.432	AT UV AH RD
4	0.98394298	64.907482	AT CU AH CR
4	0.99719686	8.854882	UV AH RD CR
4	0.99726112	8.583107	CU UV AH RD
4	0.99734420	8.231779	CU UV RD CR
4	0.99736602	8.139497	CU UV AH CR
4	0.99737298	8.110047	CU AH RD CR
4	0.99774982	6.516351	AT AH RD CR
4	0.99794352	5.697150	AT CU AH RD
4	0.99818515	4.675249	AT CU RD CR
4	0.99836034	3.934375	AT UV AH CR
4	0.99845255	3.544391	AT CU UV AH
4	0.99847154	3.464060	AT CU UV CR
4	0.99856768	3.057494	AT CU UV RD
4	0.99857478	3.027456	AT UV RD CR
5	0.99738936	10.040779	CU UV AH RD CR
5	0.99838499	5.830128	AT CU AH RD CR
5	0.99853777	5.183987	AT CU UV AH CR
5	0.99857502	5.026441	AT CU UV RD CR
5	0.99857698	5.018147	AT UV AH RD CR
5	0.99858121	5.000255	AT CU UV AH RD
6	0.99858127	7.000000	AT CU UV AH RD CR

Figure 10 *(continued from the previous page)*

parameters are deleted from the discussion either because of their insignificance or because their effect is incorporated in other parameters considered for this study. The discarded parameters include maximum and minimum temperature and humidity. Average temperature and humidity are considered to incorporate the effect of minima and maxima.

Building a regression model that includes only a subset of the available regressors involves two conflicting objectives: First, it is desirable to have a model that includes as many regressors as possible so that the information content in these factors can influence the predicted values of the dependent variable. Second, it is recommended to include as few regressors as possible because the variance of the prediction variable increases as the number of regressors increase. In this work, effort is made to find a model that is a compromise between these two objectives.

The selection of variables considered significant for the mathematical model was based on the stepwise regression methods. Evaluation of all possible regressions for determining the significant independent variables is practically not possible.[12] To overcome this burdensome computation, various methods have been developed for evaluating only a small number of subset regression models by either adding or deleting regressors one at a time. These methods are referred to as a stepwise-type procedure and are classified into forward selection, backward elimination, and stepwise regression. Stepwise uses the selection strategies in choosing the variables for the models it considers.[13]

The forward selection technique of the stepwise procedure begins with the assumption that there are no regressors in the model other than the intercept. An effort is made to find an optimal subset by inserting regressors into the model one at a time. The first regressor selected for entry into the equation is the one that has the largest simple correlation with the response variable. The chosen regressor will produce the largest value of the F-statistics for testing significance of regression. This regressor is entered if the F-statistics exceed a preselected F-value, say F_{IN} (or F-to-enter). The second regressor chosen for entry is the one that now has the largest correlation with the response variable after adjusting for the effect of the first regressor entered in the model. These correlations are referred to as partial correlation. In general, at each step, the regressor having the highest partial correlation with the response variable is added to the model if its partial F-statistics exceeds the preselected entry level F_{IN}. The procedure terminates either when the partial F-statistic at a particular step does not exceed F_{IN} or when the last candidate regressor is added to the model.[14]

TENSILE STRENGTH MATHEMATICAL MODEL

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: TS TENSILE STRENGTH

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE
MODEL	3	54548.56387834	18182.85462611
ERROR	9	85.74381397	9.52709044
CORRECTED TOTAL	12	54634.30769231	

MODEL F = 1908.54 PR > F = 0.0001

R-SQUARE	C.V.	ROOT MSE	TS MEAN
0.998431	3.5987	3.08659852	85.76923077

SOURCE	DF	TYPE I SS	F VALUE	PR > F
AT	1	17754.96599332	1863.63	0.0001
CU	1	35787.69323041	3756.41	0.0001
UV	1	1005.90465461	105.58	0.0001

SOURCE	DF	TYPE III SS	F VALUE	PR > F
AT	1	113.58388652	11.92	0.0072
CU	1	36186.53117283	3798.28	0.0001
UV	1	1005.90465461	105.58	0.0001

PARAMETER	ESTIMATE	T FOR H0: PARAMETER=0	PR > |T|	STD ERROR OF ESTIMATE
INTERCEPT	220.51495418	73.00	0.0001	3.02076942
AT	-0.58009691	-3.45	0.0072	0.16800505
CU	-0.84254882	-61.63	0.0001	0.01367105
UV	-2.11782006	-10.28	0.0001	0.20610607

Figure 11 Computer output (SAS) of general linear regression model procedure applied to LLDPE tensile strength data.

Forward selections begin with no regressors in the model and attempts to insert variables until a suitable model is obtained. Backward elimination attempts to find a good model by working in the opposite direction. It begins with calculating statistics for a model including all of the independent variables. Then, the partial F-statistic is computed for each regressor as if it were the last variable to enter the model. The smallest of these partial F-statistics is compared with a preselected value, F_{OUT} (or F-to-remove), for example, and if the smallest partial F-value is less than F_{OUT}, that regressor is removed from the model. Now, a regression model with one less independent variable is fit, the partial F-statistics for this new model calculated, and the procedure repeated. The program terminates when the smallest partial F-value is not less than the preselected cutoff value F_{OUT}. Stepwise regression is a

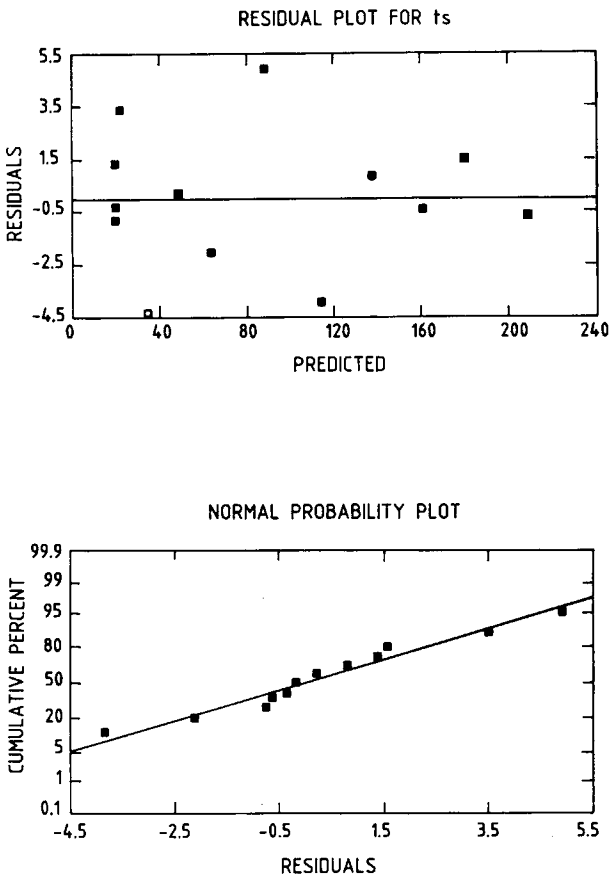


Figure 12 Residual and normal probability plot of LLDPE tensile strength (ts) model.

modification of forward selection in which at each step all regressors entered into the model previously are reassessed via their partial F-statistics. A regressor added at an early step may now be redundant because of the relationships between it and regressors now in the equation. If the partial F-statistic for a variable is less than F_{OUT}, that variable is dropped from the model. Stepwise regression requires two cutoff values, F_{IN} and F_{OUT}.[14]

The coefficient of multiple determination (R^2) has been widely used as a measure of the adequacy of a regression model. Generally, it is not straightforward to use R^2 as a criterion for choosing the number of regressors to include in the model. However, for a fixed number of variables, R^2 can be used to compare the generated models. Mallows has proposed a criterion that is related to the mean square error of the fitted values and it is called Mallow's C_p statistic.[15] Generally, small values of C_p are desirable; C_p values less than the number of independent variables represent a model with lower total errors.[15] The RSQUARE procedure of SAS was used to determine R^2 and Mallow's C_p statistic for each model. The program evaluates each combination of a dependent variable with the independent variables. If K independent variables are specified, the program evaluates each of the 2^{K-1} linear models: K of the models includes one independent variable, $K(K-1)/2$ of the model includes two independent variables, and so on. For each model evaluated, the program prints the unadjusted R^2 value and Mallow's C_p statistic.[16]

Model I

Mechanical properties of plastics are important ultimate indicators of plastic behavior when exposed to weather. Mathematically,

Degradation rate α drop in mechanical properties

Therefore, the dependence of mechanical property (tensile strength [TS]) on weather parameters is presented in a functional relationship of the form

$$\mathrm{TS} = \mathrm{F}(\mathrm{AT}, \mathrm{AH}, \mathrm{UV}, \mathrm{CU}, \mathrm{RD}, \mathrm{CR}).$$

Variable Selection

The SAS stepwise regression algorithm was used, and the results of the forward selection procedure are presented in Figure 7. In this program, cutoff value F_{IN} is specified by choosing a type I error rate, α. Therefore, the regressor with highest partial correlation with a dependent variable is added to the model if its partial F-statistic exceeds $F_{\alpha,1,n-p}$. In this work, $\alpha = .05$ to determine F_{IN}. It is shown in Figure 7, step 1, that the regressor most highly correlated with tensile strength of plastic is cumulative UV (CU). The F-statistics associated with the model using CU is $F = 178.29 > F_{.05,1,11} = 4.48$; CU is added to the equation. At step 2, the regressor having the largest partial correlation with TS (or the largest partial F statistic given that CU is in the model) is UV, and since the partial F-statistic is $F = 149.28$, which exceeds $F_{IN} = F_{.05,1,11} = 4.96$, UV is added to the model. In the third step, AT exhibits the highest partial correlation with TS. The partial F statistic is 11.92, which is larger than $F_{IN} = F_{.05,1,9} = 5.12$, and so AT is added to the model. At this point, the remaining candidate regressors are AH, RD, and CR, and for which the partial F-statistic does not exceed $F_{.05,1,8} = 5.32$, so the forward selection procedure terminates with

$$\mathrm{TS} = 220.51 - 0.58\ \mathrm{AT} - 2.12\ \mathrm{UV} - 0.84\ \mathrm{CU}$$

as the final model.

```
FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE CA

STEP 1    VARIABLE CU ENTERED          R SQUARE = 0.97945921
                                       C(P) =     11.80734240
                DF     SUM OF SQUARES  MEAN SQUARE        F     PROB>F
REGRESSION       1         7.23124189   7.23124189   524.52     0.0001
ERROR           11         0.15165042   0.01378640
TOTAL           12         7.38289231

                B VALUE     STD ERROR   TYPE II SS        F     PROB>F

INTERCEPT   0.44908868
CU          0.01089143     0.00047556   7.23124189   524.52     0.0001

BOUNDS ON CONDITION NUMBER:              1,            1
-----------------------------------------------------------------------

STEP 2    VARIABLE UV ENTERED          R SQUARE = 0.98776628
                                       C(P) =     5.39247629
                DF     SUM OF SQUARES  MEAN SQUARE        F     PROB>F
REGRESSION       2         7.29257207   3.64628603   403.71     0.0001
ERROR           10         0.09032024   0.00903202
TOTAL           12         7.38289231

                B VALUE     STD ERROR   TYPE II SS        F     PROB>F

INTERCEPT   0.29336879
UV          0.01186522     0.00455335   0.06133017     6.79     0.0262
CU          0.01061289     0.00039949   6.37450356   705.77     0.0001

BOUNDS ON CONDITION NUMBER:      1.077119,      4.308475
-----------------------------------------------------------------------

NO OTHER VARIABLES MET THE 0.0500 SIGNIFICANCE LEVEL FOR ENTRY

SUMMARY OF FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE CA

      VARIABLE  NUMBER      PARTIAL       MODEL
STEP  ENTERED       IN         R**2        R**2           C(P)

   1  CU             1       0.9795      0.9795        11.8073
   2  UV             2       0.0083      0.9878         5.3925

      VARIABLE
STEP  ENTERED             F      PROB>F   LABEL

   1  CU           524.5199      0.0001   CUMULATIVE UV RADIATION
   2  UV             6.7903      0.0262   UV RADIATION
-----------------------------------------------------------------------
```

Figure 13 Computer output (SAS) stepwise forward selection procedure applied to LLDPE carbonyl data.

The backward elimination algorithm of SAS was also used, and the results are presented in Figure 8. In this run, cutoff value F_{OUT} is chosen as $\alpha = .05$. Thus, a regressor is dropped if its partial F-statistic is less than $F_{.05,1,n-p}$. Step 0 shows the results of fitting the full model. The smallest partial F-value is $F = 0.00$, and it is associated with CR. Thus, since $F = 0.00 < F_{OUT} = F_{.05,1,6} = 5.99$, CR is removed from the model. At step 1, the results of fitting a five-variable model involving (AT, AH, RD, UV, CU) are presented. The smallest partial F-value in this model, $F = 0.07$, is associated with AH. Since

```
BACKWARD ELIMINATION PROCEDURE FOR DEPENDENT VARIABLE CA

STEP 0    ALL VARIABLES ENTERED        R SQUARE = 0.99407686
                                       C(P) =     7.00000000
                 DF   SUM OF SQUARES  MEAN SQUARE        F    PROB>F
REGRESSION        6       7.33916243   1.22319374   167.83    0.0001
ERROR             6       0.04372988   0.00728831
TOTAL            12       7.38289231

                B VALUE    STD ERROR    TYPE II SS        F    PROB>F
INTERCEPT   -0.00504506
AT           0.00676125   0.00704551    0.00671206     0.92    0.3743
AH           0.00221525   0.00522462    0.00131028     0.18    0.6863
RD          -0.00495549   0.00344885    0.01504707     2.06    0.2008
UV           0.15926972   0.10079886    0.01819626     2.50    0.1652
CR           0.00540511   0.00456632    0.01021182     1.40    0.2813
CU          -0.14677233   0.13252997    0.00893896     1.23    0.3105

BOUNDS ON CONDITION NUMBER:      147326.3,      1773238
-------------------------------------------------------------------
STEP 1    VARIABLE AH REMOVED          R SQUARE = 0.99389939
                                       C(P) =     5.17977833
                 DF   SUM OF SQUARES  MEAN SQUARE        F    PROB>F
REGRESSION        5       7.33785215   1.46757043   228.09    0.0001
ERROR             7       0.04504016   0.00643431
TOTAL            12       7.38289231

                B VALUE    STD ERROR    TYPE II SS        F    PROB>F
INTERCEPT    0.12690454
AT           0.00598345   0.00639156    0.00563887     0.88    0.3804
RD          -0.00583890   0.00258235    0.03289531     5.11    0.0582
UV           0.18560547   0.07459217    0.03983795     6.19    0.0417
CR           0.00673744   0.00311307    0.03013796     4.68    0.0672
CU          -0.18531251   0.09062080    0.02690640     4.18    0.0801

BOUNDS ON CONDITION NUMBER:      77803.07,      780913.6
-------------------------------------------------------------------
STEP 2    VARIABLE AT REMOVED          R SQUARE = 0.99313561
                                       C(P) =     3.95346438
                 DF   SUM OF SQUARES  MEAN SQUARE        F    PROB>F
REGRESSION        4       7.33221329   1.83305332   289.36    0.0001
ERROR             8       0.05067902   0.00633488
TOTAL            12       7.38289231

                B VALUE    STD ERROR    TYPE II SS        F    PROB>F
INTERCEPT    0.22089466
RD          -0.00432837   0.00200054    0.02965469     4.68    0.0624
UV           0.14389686   0.05936060    0.03722583     5.88    0.0416
CR           0.00466888   0.00217586    0.02916771     4.60    0.0642
CU          -0.12503730   0.06327470    0.02473758     3.90    0.0836
BOUNDS ON CONDITION NUMBER:      38526.95,      310085.9
-------------------------------------------------------------------
```

Figure 14 Computer output (SAS) of stepwise backward elimination procedure applied to LLDPE carbonyl data.

$F = 0.07 < F_{OUT} = F_{.05,1,7} = 5.59$, AH is removed from the model. Similarly, in step 2, RD is removed. At step 3, the results of fitting the three-variable model involving (AT, UV, CU) are shown. The smallest partial F-statistic in this model is $F = 11.92$, associated with AT, and since this exceeds $F_{.05,1,9}$

STEP 3 VARIABLE CU REMOVED R SQUARE = 0.98978495

C(P) = 5.34760840

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	3	7.30747570	2.43582523	290.68	0.0001
ERROR	9	0.07541660	0.00837962		
TOTAL	12	7.38289231			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.28988999				
RD	-0.00242046	0.00201517	0.01208924	1.44	0.2604
UV	0.08188589	0.05795221	0.01673025	2.00	0.1913
CR	0.00036922	0.00001374	6.05102943	722.11	0.0001

BOUNDS ON CONDITION NUMBER: 190.1819, 1138.214

STEP 4 VARIABLE RD REMOVED R SQUARE = 0.98814748

C(P) = 5.00632471

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	2	7.29538646	3.64769323	416.85	0.0001
ERROR	10	0.08750585	0.00875058		
TOTAL	12	7.38289231			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	0.28718989				
UV	0.01247726	0.00447577	0.06800499	7.77	0.0192
CR	0.00036473	0.00001351	6.37731796	728.79	0.0001

BOUNDS ON CONDITION NUMBER: 1.074197, 4.296789

ALL VARIABLES IN THE MODEL ARE SIGNIFICANT AT THE 0.0500 LEVEL.

SUMMARY OF
BACKWARD ELIMINATION PROCEDURE FOR DEPENDENT VARIABLE CA

STEP	VARIABLE REMOVED	NUMBER IN	PARTIAL R**2	MODEL R**2	C(P)
1	AH	5	0.0002	0.9939	5.17978
2	AT	4	0.0008	0.9931	3.95346
3	CU	3	0.0034	0.9898	5.34761
4	RD	2	0.0016	0.9881	5.00632

STEP	VARIABLE REMOVED	F	PROB>F	LABEL
1	AH	0.1798	0.6863	MONTHLY AVERAGE HUMIDITY
2	AT	0.8764	0.3804	MONTHLY AVERAGE TEMPERATURE
3	CU	3.9050	0.0836	CUMULATIVE UV RADIATION
4	RD	1.4427	0.2604	TOTAL SOLAR RADIATION

Figure 14 *(continued from the previous page)*

= 5.12, no further regressor can be removed from the model. Therefore, backward elimination terminates, yielding the final model

$$TS = 220.51 - .58\ AT - 2.12\ UV - .84\ CU$$

Figure 9 presents the results of using the SAS stepwise regression algorithm. The level for either adding or removing a regressor is specified as 0.05. At step 1, the procedure begins with no variables in the model and tries to add CU. Since the partial F-statistic at this step exceeds $F_{IN} = F_{.05,1,11} = 4.48$, CU is added to the model. At step 2, UV is added to the model, and at step 3, AT is incorporated in the model. At this point, the remaining candidate regressors are (RD, AH, CR), which cannot be added because its partial F-value does not exceed preset limits. Therefore, stepwise regression terminates with the model

$$TS = 220.51 - .58\ AT - 2.12\ UV - .84\ CU$$

It is noticed that the model developed by forward

```
STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE CA

STEP 1    VARIABLE CU ENTERED          R SQUARE = 0.97945921
                                       C(P) =     11.80734240
              DF   SUM OF SQUARES  MEAN SQUARE        F    PROB>F
REGRESSION     1       7.23124189   7.23124189   524.52    0.0001
ERROR         11       0.15165042   0.01378640
TOTAL         12       7.38289231

             B VALUE    STD ERROR    TYPE II SS        F    PROB>F
INTERCEPT  0.44908868
CU         0.01089143  0.00047556    7.23124189   524.52    0.0001

BOUNDS ON CONDITION NUMBER:                 1,            1
------------------------------------------------------------------

STEP 2    VARIABLE UV ENTERED          R SQUARE = 0.98776628
                                       C(P) =      5.39247629
              DF   SUM OF SQUARES  MEAN SQUARE        F    PROB>F
REGRESSION     2       7.29257207   3.64628603   403.71    0.0001
ERROR         10       0.09032024   0.00903202
TOTAL         12       7.38289231

             B VALUE    STD ERROR    TYPE II SS        F    PROB>F
INTERCEPT  0.29336879
UV         0.01186522  0.00455335    0.06133017     6.79    0.0262
CU         0.01061289  0.00039949    6.37450356   705.77    0.0001

BOUNDS ON CONDITION NUMBER:      1.077119,      4.308475
------------------------------------------------------------------

NO OTHER VARIABLES MET THE 0.0500 SIGNIFICANCE LEVEL FOR ENTRY

SUMMARY OF
STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE CA

        VARIABLE          NUMBER   PARTIAL    MODEL
STEP  ENTERED   REMOVED       IN      R**2     R**2          C(P)
   1  CU                       1    0.9795   0.9795       11.8073
   2  UV                       2    0.0083   0.9878        5.3925

        VARIABLE
STEP  ENTERED   REMOVED            F            PROB>F
   1  CU                    524.5199            0.0001
   2  UV                      6.7903            0.0262

        VARIABLE
STEP  ENTERED   REMOVED    LABEL
   1  CU                   CUMULATIVE UV RADIATION
   2  UV                   UV RADIATION
```

Figure 15 Computer output (SAS) of stepwise regression procedure applied to LLDPE carbonyl data.

selection, backward elimination, and stepwise regression techniques has resulted in the same intercept, independent variables, and the coefficients of independent variables.

R^2 and Mallow's C_p values were determined using the RSQUARE procedure of the SAS software package, and the results are shown in Figure 10. It is obvious from the table that the best combination

```
N=13          REGRESSION MODELS FOR DEPENDENT VARIABLE: CA
              MODEL: MODEL1
```

NUMBER IN MODEL	R-SQUARE	C(P)	VARIABLES IN MODEL
1	0.12435079	878.012	UV
1	0.12548598	876.862	AH
1	0.13525087	866.971	RD
1	0.18936007	812.160	AT
1	0.97893633	12.337015	CR
1	0.97945921	11.807342	CU
2	0.17018348	833.585	UV RD
2	0.19244490	811.035	AT UV
2	0.19427626	809.180	AT RD
2	0.38343679	617.564	UV AH
2	0.40230088	598.455	AH RD
2	0.67246416	324.786	AT AH
2	0.97896469	14.308279	AH CR
2	0.97957215	13.692939	CU AH
2	0.98041009	12.844124	CU CR
2	0.98094481	12.302463	AT CR
2	0.98101096	12.235461	AT CU
2	0.98720821	5.957790	CU RD
2	0.98751887	5.643099	RD CR
2	0.98776628	5.392476	CU UV
2	0.98814748	5.006325	UV CR
3	0.21219935	793.024	AT UV RD
3	0.44469444	557.512	UV AH RD
3	0.67257128	326.678	AT UV AH
3	0.67283251	326.413	AT AH RD
3	0.98102544	14.220796	AT CU CR
3	0.98262385	12.601638	CU AH CR
3	0.98528621	9.904726	AT AH CR
3	0.98557321	9.614007	AT CU AH
3	0.98809344	7.061071	CU RD CR
3	0.98826054	6.891804	AT CU RD
3	0.98839248	6.758148	AT RD CR
3	0.98889384	6.250283	AT CU UV
3	0.98910196	6.039460	AT UV CR
3	0.98911894	6.022265	CU UV CR
3	0.98918490	5.955448	CU UV RD
3	0.98978495	5.347608	UV RD CR
3	0.99112755	3.987586	AH RD CR
3	0.99123967	3.874009	CU AH RD
3	0.99173196	3.375336	UV AH CR
3	0.99176567	3.341189	CU UV AH

Figure 16 Computer output (SAS) of RSQUARE and Mallow's Cp procedure applied LLDPE carbonyl data.

of R^2 and Mallow's C_p is for three-variable model with AT, CU, and UV as independent variables. The value of R^2 is .998, which is extremely good. The C_p value is 1.64, which is minimum of all combinations evaluated and also less than independent variables considered.

4	0.67964305	321.514	AT UV AH RD
4	0.98647559	10.699915	AT CU AH CR
4	0.98850341	8.645783	AT CU RD CR
4	0.98944378	7.693211	AT CU UV CR
4	0.98981725	7.314887	AT CU UV RD
4	0.99025496	6.871496	AT UV RD CR
4	0.99134695	5.765336	CU AH RD CR
4	0.99146015	5.650669	AT AH RD CR
4	0.99154803	5.561649	AT CU AH RD
4	0.99176875	5.338070	CU UV AH CR
4	0.99201954	5.084024	AT UV AH CR
4	0.99203873	5.064580	AT CU UV AH
4	0.99232020	4.779458	CU UV AH RD
4	0.99246396	4.633832	UV AH RD CR
4	0.99313561	3.953464	CU UV RD CR
5	0.99161221	7.496636	AT CU AH RD CR
5	0.99203876	7.064547	AT CU UV AH CR
5	0.99269369	6.401123	AT CU UV AH RD
5	0.99286610	6.226479	AT UV AH RD CR
5	0.99316773	5.920934	CU UV AH RD CR
5	0.99389939	5.179778	AT CU UV RD CR
6	0.99407686	7.000000	AT CU UV AH RD CR

Figure 16 *(continued from the previous page)*

Regression Analysis

A multiple regression model was developed using the SAS algorithm for the best subset regressor variables. The model incorporates these independent variables that are statistically selected in the previous section (UV, AT, and CU) and the TS as a dependent variable. The results are presented in Figure 11. This figure shows that the regression model is very significant and has a coefficient of variance (CV) of 3.6 and root mean square error of 3.09. The developed model is same as the one proposed by the different variable selection techniques:

$$\text{TS} = 220.52 - 0.58\ \text{AT} - 0.84\ \text{CU} - 2.12\ \text{UV}$$

Residual Analysis

The functional form of the model presented earlier was used to predict the tensile strength (TS), and the results were compared to find the residuals. Residuals are defined as

$$e_i = y_i - y'_i, \quad i = 1, 2 \cdots n$$

where y_i is an observation and y'_i is the corresponding fitted value. Since a residual may be viewed as the duration between the data and the fit, it is a measure of the variability not explained by the model.

The adequacy of the model can be viewed from the plot of residual against predicted values of TS (Fig. 12). This plot indicates that the residuals can be contained in a horizontal band. The scatter indicates no trend inequality of variance, and, therefore, there is no obvious model defect.

Although small departures from normality do not affect the model greatly, gross nonnormality is potentially more serious. A very simple method of checking the normality assumption is to plot the residual on normal probability paper. Figure 12 also shows the normal probability plot of residuals and the cumulative percent, which shows a reasonably straight line. Slight deviation from the straight line can be attributed to small number of observations.[17]

Model II

Growth in the carbonyl group is an important indication of the extent of degradation in polymers. In this section, a linear multiple regression model will be developed with carbonyl growth as a dependent variable and weather parameters as independent variables.

```
CARBONYL GROUP MATHEMATICAL MODEL

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: CA          CARBONYL GROWTH

SOURCE            •      DF     SUM OF SQUARES         MEAN SQUARE

MODEL                     4         7.33221329          1.83305332

ERROR                     8         0.05067902          0.00633488

CORRECTED TOTAL          12         7.38289231

MODEL F =            289.36                        PR > F = 0.0001

R-SQUARE               C.V.           ROOT MSE             CA MEAN

0.993136             5.0720         0.07959195          1.56923077

SOURCE                   DF         TYPE I SS    F VALUE    PR > F

CU                        1         7.23124189   1141.50    0.0001
UV                        1         0.06133017      9.68    0.0144
RD                        1         0.01047351      1.65    0.2345
CR                        1         0.02916771      4.60    0.0642

SOURCE                   DF       TYPE III SS    F VALUE    PR > F

CU                        1         0.02473758      3.90    0.0836
UV                        1         0.03722583      5.88    0.0416
RD                        1         0.02965469      4.68    0.0624
CR                        1         0.02916771      4.60    0.0642

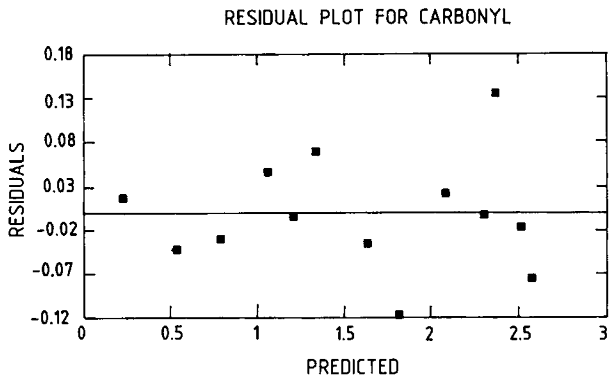
                                   T FOR H0:    PR > |T|    STD ERROR OF
PARAMETER          ESTIMATE     PARAMETER=0                    ESTIMATE

INTERCEPT        0.22089466            3.03      0.0164      0.07294617
CU              -0.12503730           -1.98      0.0836      0.06327470
UV               0.14389686            2.42      0.0416      0.05936060
RD              -0.00432837           -2.16      0.0624      0.00200054
CR               0.00466888            2.15      0.0642      0.00217586
--------------------------------------------------------------------
```

Figure 17 Computer output (SAS) of general linear model procedure applied to LLDPE carbonyl data.

Variable Selection

The same procedures as used earlier for model I will be used. Figure 13 shows the results obtained when an SAS forward selection algorithm was applied to the data. In this program, the cutoff value $\alpha = .05$ is specified. It is indicated in the results that the most highly correlated regressor with carbonyl growth is CU, and since the statistics associated with the model using CU is $F = 524.8 >$, which is greater



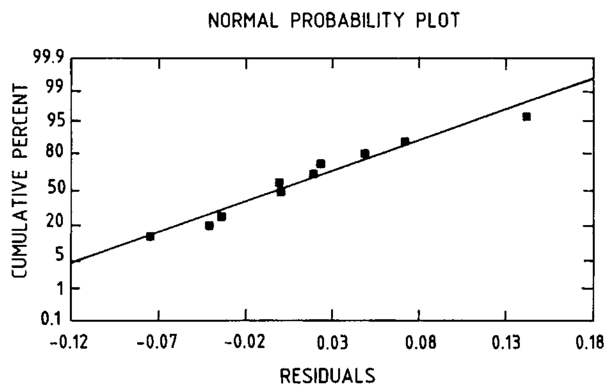


Figure 18 Residual and normal probability plot of LLDPE carbonyl model.

than $F_{.05,1,11} = 4.48$, CU is added to the equation. At step 2, the regressor having the largest partial correlation with carbonyl growth is UV, and since the partial F-statistic for this regressor is 6.79, which exceeds $F_{IN} = F_{.05,1,10} = 4.96$, UV is added to the model. At this point, the partial F-statistic $F_{IN} = F_{.05,1,9} = 5.12$ exceeds the F-value of all regressors, so the forward selection terminates with

$$CA = 0.29 + .012\ UV + .01\ CU$$

as the final model.

The results of the backward elimination procedure for dependent variable CA are presented in Figure 14. Step 0 shows the results of fitting the full model. The smallest partial F-value is $F = 0.18$, and it is associated with AH. Thus, since $F = 0.18 < F_{OUT} = F_{.05,1,6} = 5.99$, AH is removed from the model. At step 1, the results of fitting the five variables involving (AT, RD, UV, CR, CU) are shown. The smallest partial F-value in this model, $F = 0.88 < F_{OUT} = F_{.05,1,7} = 5.59$, AT is removed from the model. At step 2, the results of fitting the four-variable model is shown. The smallest partial F-statistic in this model is $F = 3.90$, associated with CU, and since this is less than $F_{OUT} = F_{.05,1,8} = 5.32$, CU is removed from the model. Similarly, RD is also removed, and, finally, backward elimination terminates, yielding the final model

$$CA = 0.287 + 0.12\ UV + .0004\ CR$$

The SAS stepwise regression algorithm was used for stepwise regression, and the results are presented in Figure 15. At step 1, the procedure begins with no variables in the model and tries to add CU. Since the partial F-statistic at this step exceeds $F_{IN} = F_{.05,1,11} = 3.23$, CU is added to the model. Similarly, UV is also added, and for the other candidate regressor, F-values were found lower than F_{IN}. Therefore, the regression terminates with the model

$$CA = .29 + .012\ UV + .01\ CU$$

R^2 and Mallow's C_p was determined for each combination of independent variable using the SAS algorithm. The results are presented in Figure 16. Analyzing the results indicates that the R^2 value is within reasonable limits. Mallow's C_p is less than the number of parameters only at one point when the number of parameters is 4 (CU, UV, RD, CR), $C_p = 3.95$, and $R^2 = 0.993$. The results indicated by forward, backward, and stepwise do not show a common selection trend. R^2 and Mallow's C_p results are also different, which is not unusual.[18] In order to have the model that includes all those independent variables that are suggested by different methods, all the variables selected were incorporated in the final model. These independent variables are CU, UV, RD, and CR.

Regression Analysis

Based on weather parameters selected in the previous section, a regression model was developed for growth in carbonyl peaks as a function of these variables. The results of the general linear models procedure of SAS are shown in Figure 17. The figure indicates a coefficient of variance (CV) = 5.07 and root mean square error of .08. Both of these values

```
FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE CY

STEP 1    VARIABLE CR ENTERED          R SQUARE = 0.86096430
                                       C(P) =    71.54071626
                DF   SUM OF SQUARES  MEAN SQUARE        F    PROB>F

REGRESSION       1    379.86804532   379.8680453    68.12    0.0001
ERROR           11     61.34426237     5.5767511
TOTAL           12    441.21230769

                B VALUE     STD ERROR    TYPE II SS        F    PROB>F

INTERCEPT  37.05077773
CR          0.00271600    0.00032908   379.8680453    68.12    0.0001

BOUNDS ON CONDITION NUMBER:             1,             1
-----------------------------------------------------------------------
STEP 2    VARIABLE AT ENTERED          R SQUARE = 0.98366920
                                       C(P) =     2.46011789
                DF   SUM OF SQUARES  MEAN SQUARE        F    PROB>F

REGRESSION       2    434.00695889   217.0034794   301.17    0.0001
ERROR           10      7.20534881     0.7205349
TOTAL           12    441.21230769

                B VALUE     STD ERROR    TYPE II SS        F    PROB>F

INTERCEPT  43.22679719
AT         -0.28667276    0.03307190    54.1389136    75.14    0.0001
CR          0.00316118    0.00012896   432.9807235   600.92    0.0001

BOUNDS ON CONDITION NUMBER:      1.188515,      4.754059
-----------------------------------------------------------------------
NO OTHER VARIABLES MET THE 0.0500 SIGNIFICANCE LEVEL FOR ENTRY

SUMMARY OF FORWARD SELECTION PROCEDURE FOR DEPENDENT VARIABLE CY

      VARIABLE  NUMBER      PARTIAL       MODEL
STEP  ENTERED       IN         R**2        R**2          C(P)

   1  CR             1       0.8610      0.8610       71.5407
   2  AT             2       0.1227      0.9837        2.4601

      VARIABLE
STEP  ENTERED            F       PROB>F    LABEL

   1  CR           68.1164       0.0001    CUMULATIVE RADIATION
   2  AT           75.1371       0.0001    MONTHLY AVERAGE TEMPERATURE
-----------------------------------------------------------------------
```

Figure 19 Computer output (SAS) of stepwise forward selection procedure applied to LLDPE crystallinity data.

indicate that the developed model is reliable. The developed model is

$$CA = 0.22 - 0.125\ CU + 0.144\ UV - 0.004\ RD + 0.005\ CR$$

Residual Analysis

The adequacy of the model is very well exhibited by the plot of residuals against predicted values (Fig.

```
BACKWARD ELIMINATION PROCEDURE FOR DEPENDENT VARIABLE CY

STEP 0    ALL VARIABLES ENTERED        R SQUARE = 0.98964233
                                       C(P) =     7.00000000
                DF   SUM OF SQUARES  MEAN SQUARE        F    PROB>F
REGRESSION       6     436.64237597  72.77372933    95.55    0.0001
ERROR            6       4.56993172   0.76165529
TOTAL           12     441.21230769

               B VALUE   STD ERROR    TYPE II SS        F    PROB>F
INTERCEPT   45.6936070
AT          -0.2900256  0.07202419  12.35021328     16.21    0.0069
AH          -0.0236481  0.05340976   0.14931773      0.20    0.6734
RD           0.0099791  0.03525655   0.06101908      0.08    0.7867
UV          -0.3794213  1.03043677   0.10326642      0.14    0.7254
CR          -0.0140199  0.04668013   0.06870414      0.09    0.7741
CU           0.5024750  1.35481451   0.10476778      0.14    0.7235

BOUNDS ON CONDITION NUMBER:     147326.3,      1773238
----------------------------------------------------------------
STEP 1    VARIABLE RD REMOVED          R SQUARE = 0.98950403
                                       C(P) =     5.08011377
                DF   SUM OF SQUARES  MEAN SQUARE        F    PROB>F
REGRESSION       5     436.58135689  87.31627138   131.98    0.0001
ERROR            7       4.63095080   0.66156440
TOTAL           12     441.21230769

               B VALUE   STD ERROR    TYPE II SS        F    PROB>F
INTERCEPT   46.0543931
AT          -0.2834314  0.06351644  13.17333736     19.91    0.0029
AH          -0.0327807  0.03966712   0.45180114      0.68    0.4358
UV          -0.0883142  0.05903898   1.48032110      2.24    0.1783
CR          -0.0031058  0.02451985   0.01061410      0.02    0.9028
CU           0.1858933  0.71252425   0.04502982      0.07    0.8017
BOUNDS ON CONDITION NUMBER:    46799.17,     467955.9
----------------------------------------------------------------
STEP 2    VARIABLE CR REMOVED          R SQUARE = 0.98947997
                                       C(P) =     3.09404934
                DF   SUM OF SQUARES  MEAN SQUARE        F    PROB>F
REGRESSION       4     436.57074279  109.1426857   188.11    0.0001
ERROR            8       4.64156490    0.5801956
TOTAL           12     441.21230769

               B VALUE   STD ERROR    TYPE II SS        F    PROB>F
INTERCEPT   46.1334537
AT          -0.2826638  0.05921088   13.2224465     22.79    0.0014
AH          -0.0348963  0.03369411    0.6223367      1.07    0.3306
UV          -0.0855359  0.05133178    1.6110132      2.78    0.1342
CU           0.0956441  0.00496542  215.2676887    371.03    0.0001

BOUNDS ON CONDITION NUMBER:    4.731185,     50.00516
----------------------------------------------------------------
```

Figure 20 Computer output (SAS) of stepwise backward elimination procedure applied to LLDPE crystallinity data.

18). The scatter indicates no trends or curvature, and inequality of variance also indicates a reasonably good straight line. A slight deviation from a straight line can be attributed to a small number of observations. This implies that there are no obvious defects in the developed model.

STEP 3 VARIABLE AH REMOVED R SQUARE = 0.98806946
C(P) = 1.91113375

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	3	435.94840613	145.3161354	248.46	0.0001
ERROR	9	5.26390156	0.5848780		
TOTAL	12	441.21230769			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	43.3990335				
AT	-0.2388827	0.04162697	19.2612688	32.93	0.0003
UV	-0.0927080	0.05106734	1.9275835	3.30	0.1028
CU	0.0918709	0.00338730	430.2416584	735.61	0.0001

BOUNDS ON CONDITION NUMBER: 2.319667, 16.82329

STEP 4 VARIABLE UV REMOVED R SQUARE = 0.98370062
C(P) = 2.44191575

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	2	434.02082264	217.0104113	301.76	0.0001
ERROR	10	7.19148506	0.7191485		
TOTAL	12	441.21230769			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	43.3128103				
AT	-0.2915207	0.03311917	55.7184092	77.48	0.0001
CU	0.0921002	0.00375343	432.9945872	602.09	0.0001

BOUNDS ON CONDITION NUMBER: 1.194212, 4.776849

ALL VARIABLES IN THE MODEL ARE SIGNIFICANT AT THE 0.0500 LEVEL.

SUMMARY OF
BACKWARD ELIMINATION PROCEDURE FOR DEPENDENT VARIABLE CY

STEP	VARIABLE REMOVED	NUMBER IN	PARTIAL R**2	MODEL R**2	C(P)
1	RD	5	0.0001	0.9895	5.08011
2	CR	4	0.0000	0.9895	3.09405
3	AH	3	0.0014	0.9881	1.91113
4	UV	2	0.0044	0.9837	2.44192

STEP	VARIABLE REMOVED	F	PROB>F	LABEL
1	RD	0.0801	0.7867	TOTAL SOLAR RADIATION
2	CR	0.0160	0.9028	CUMULATIVE RADIATION
3	AH	1.0726	0.3306	MONTHLY AVERAGE HUMIDITY
4	UV	3.2957	0.1028	UV RADIATION

Figure 20 *(continued from the previous page)*

Model III

The percent crystallinity (CY) of polyethylene is observed to increase with the exposure of polymer to the natural environment. In this section, a regression model will be developed to present the correlation between crystallinity and weather parameters.

Variables Selection

The SAS forward selection algorithm was used, and results are presented in Figure 19. The cutoff value

```
STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE CY

STEP 1    VARIABLE CR ENTERED         R SQUARE = 0.86096430
                                      C(P) =     71.54071626
                DF   SUM OF SQUARES  MEAN SQUARE          F    PROB>F
REGRESSION       1     379.86804532  379.8680453      68.12    0.0001
ERROR           11      61.34426237    5.5767511
TOTAL           12     441.21230769

               B VALUE    STD ERROR    TYPE II SS          F    PROB>F
INTERCEPT   37.0507777
CR           0.0027160   0.00032908   379.8680453      68.12    0.0001

BOUNDS ON CONDITION NUMBER:           1,              1
---------------------------------------------------------------------

STEP 2    VARIABLE AT ENTERED         R SQUARE = 0.98366920
                                      C(P) =      2.46011789
                DF   SUM OF SQUARES  MEAN SQUARE          F    PROB>F
REGRESSION       2     434.00695889  217.0034794     301.17    0.0001
ERROR           10       7.20534881    0.7205349
TOTAL           12     441.21230769

               B VALUE    STD ERROR    TYPE II SS          F    PROB>F
INTERCEPT   43.2267972
AT          -0.2866728   0.03307190    54.1389136      75.14    0.0001
CR           0.0031612   0.00012896   432.9807235     600.92    0.0001

BOUNDS ON CONDITION NUMBER:     1.188515,      4.754059
---------------------------------------------------------------------
NO OTHER VARIABLES MET THE 0.0500 SIGNIFICANCE LEVEL FOR ENTRY

SUMMARY OF
STEPWISE REGRESSION PROCEDURE FOR DEPENDENT VARIABLE CY

           VARIABLE          NUMBER   PARTIAL    MODEL
STEP  ENTERED   REMOVED          IN      R**2     R**2          C(P)
   1  CR                          1    0.8610   0.8610       71.5407
   2  AT                          2    0.1227   0.9837        2.4601

           VARIABLE
STEP  ENTERED   REMOVED          F              PROB>F
   1  CR                   68.1164              0.0001
   2  AT                   75.1371              0.0001

           VARIABLE
STEP  ENTERED   REMOVED    LABEL
   1  CR                   CUMULATIVE RADIATION
   2  AT                   MONTHLY AVERAGE TEMPERATURE
---------------------------------------------------------------------
```

Figure 21 Computer output (SAS) of stepwise regression procedure applied to LLDPE crystallinity data.

$\alpha = 0.05$ is preset, similar to the earlier two models. The most highly correlated with CY is CR, and since the F-statistic associated with the model using CR ($F = 68.12$) is greater than $F_{.05,1,11} = 4.48$, CR is added to the equation. In step 2, the regressor having the largest partial correlation with percent crystal-

N=13 REGRESSION MODELS FOR DEPENDENT VARIABLE: CY
MODEL: MODEL1

NUMBER IN MODEL	R-SQUARE	C(P)	VARIABLES IN MODEL
1	0.00039603	570.051	RD
1	0.00132731	569.512	UV
1	0.00232594	568.933	AT
1	0.31110677	390.063	AH
1	0.85741582	73.596[illegible]31	CU
1	0.86096430	71.540716	CR
2	0.00902399	567.053	AT RD
2	0.01293424	564.788	AT UV
2	0.04885118	543.982	UV RD
2	0.33948825	375.622	UV AH
2	0.34663948	371.479	AH RD
2	0.47960128	294.457	AT AH
2	0.92059458	38.998037	CU AH
2	0.92151646	38.464013	AH CR
2	0.92208489	38.134730	CU CR
2	0.94441413	25.199829	CU UV
2	0.94535850	24.652772	UV CR
2	0.94594643	24.312197	CU RD
2	0.94700637	23.698194	RD CR
2	0.98366920	2.460118	AT CR
2	0.98370062	2.441916	AT CU
3	0.05067191	544.928	AT UV RD
3	0.40508654	339.622	UV AH RD
3	0.49725457	286.231	AT AH RD
3	0.50157951	283.725	AT UV AH
3	0.93185543	34.474843	CU AH CR
3	0.94646894	26.009517	CU UV RD
3	0.94769519	25.299177	UV RD CR
3	0.95137817	23.165693	CU UV CR
3	0.95405094	21.617408	CU RD CR
3	0.95951153	18.454195	CU UV AH
3	0.95959386	18.406504	UV AH CR
3	0.95980429	18.284602	CU AH RD
3	0.96000494	18.168372	AH RD CR
3	0.98370065	4.441902	AT CU CR
3	0.98582864	3.209197	AT CU AH
3	0.98599912	3.110443	AT AH CR
3	0.98774576	2.098645	AT RD CR
3	0.98781682	2.057484	AT UV CR
3	0.98794270	1.984565	AT CU RD
3	0.98806946	1.911134	AT CU UV

Figure 22 Computer output (SAS) of RSQUARE and Mallow's Cp applied to LLDPE crystallinity data.

linity is AT, and since the partial F-statistic for this regressor is 75.14, which exceeds $F_{IN} = F_{.05,1,10} = 4.96$, AT is added to the model. At this point, the partial F-statistic $F_{IN} = F_{.05,1,9} = 5.12$ exceeds F-values of all other regressors, so the forward selection terminates with the model

$$CY = 43.23 - 0.287\ AT + 0.003\ CR.$$

The results of the backward elimination procedure

4	0.52048117	274.776	AT UV AH RD
4	0.95964689	20.375782	CU UV AH CR
4	0.95980435	20.284568	CU UV AH RD
4	0.96002108	20.159024	UV AH RD CR
4	0.96039682	19.941363	CU AH RD CR
4	0.96097146	19.608484	CU UV RD CR
4	0.98614891	5.023672	AT CU AH CR
4	0.98782073	4.055219	AT UV RD CR
4	0.98810361	3.891352	AT CU UV RD
4	0.98816026	3.858535	AT CU RD CR
4	0.98848003	3.673297	AT CU UV CR
4	0.98938560	3.148716	AT AH RD CR
4	0.98940197	3.139235	AT UV AH CR
4	0.98940791	3.135795	AT CU AH RD
4	0.98947997	3.094049	AT CU UV AH
5	0.96165079	21.214964	CU UV AH RD CR
5	0.98930390	5.196044	AT CU UV RD CR
5	0.98940488	5.137553	AT UV AH RD CR
5	0.98940828	5.135582	AT CU AH RD CR
5	0.98948661	5.090204	AT CU UV AH RD
5	0.98950403	5.080114	AT CU UV AH CR
6	0.98964233	7.000000	AT CU UV AH RD CR

Figure 22 *(continued from the previous page)*

is presented in Figure 20. Step 0 shows the fitting of the full model. The smallest partial F-value is $0.08 < F_{OUT} = F_{.05,1,6} = 5.99$; RD is removed from the model. At step 1, the results of fitting the five variables involved (AT, AH, UV, CR, CU) are shown. The smallest partial F-value in this model is $F = 0.02$, associated with CR. Since $F = 0.02$ is less than $F_{OUT} = F_{.05,1,7} = 5.59$, CR is removed from the model. At step 2, the results of fitting the four-variable model is shown. The smallest partial F-statistic in this model is 1.07, associated with AH, and since this is less than $F_{OUT} = F_{.05,1,8} = 5.32$, AH is removed from the model. Similarly, in step 4, UV is removed and, finally, a backward elimination procedure terminates, yielding the final model:

$$CY = 43.31 - 0.29 \text{ AT} + .092 \text{ CU}$$

It is worth mentioning that the intercept and coefficient of AT in backward elimination is close to the values obtained by the forward selection procedure.

The stepwise regression results are shown in Figure 21. As shown in step 1, there are no variables and the CR entered the model; since the partial F-statistic at this step exceeds $F_{IN} = F_{.05,1,11} = 3.23$, CR is added to the model. Similarly, in step 2, the F-statistic favors the addition of AT in the model. Finally, the program is terminated as the F-value of the regressors was lower than F_{IN}, thereby terminating the stepwise algorithm with the final model:

$$CY = 43.2 - 0.29 \text{ AT} + 0.003 \text{ CR}$$

R^2 and Mallow's C_p are presented in Figure 22. As indicated in the figure, there is more than one instance when C_p is less than the number of parameters. Therefore, those independent variables suggested by stepwise procedures, AT, UV, CU, and CR, are selected for the model. A preliminary residual analysis was carried out, and it was observed that the scatter of residual is indicating a slight trend. In addition to this, normal probability was not exhibiting a straight-line behavior. Different combinations were used, and it was found that the best fit is obtained by considering AT, CU, and CR as independent variables.

Regression Analysis

A multiple linear regression model was developed for percent crystallinity change with weather parameters with AT, CU, and CR as independent variables. The results of the general linear model procedure of SAS are shown in Figure 23. The figure indicates a coefficient of variance (CV) = 1.98 and root mean square error of 0.89. Both of these values

```
PERCENT CRYSTALLINITY MATHEMATICAL MODEL

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: CY          % CRYSTALLINITY

SOURCE                  DF      SUM OF SQUARES        MEAN SQUARE

MODEL                    3        434.02083298       144.67361099

ERROR                    9          7.19147471         0.79905275

CORRECTED TOTAL         12        441.21230769


MODEL F =           181.06                        PR > F = 0.0001


R-SQUARE            C.V.              ROOT MSE            CY MEAN

0.983701           1.9797            0.89389750        45.15384615


SOURCE                  DF         TYPE I SS     F VALUE    PR > F

CU                       1      378.30241340      473.44    0.0001
AT                       1       55.71840924       69.73    0.0001
CR                       1        0.00001035        0.00    0.9972


SOURCE                  DF       TYPE III SS     F VALUE    PR > F

CU                       1        0.01387410        0.02    0.8981
AT                       1       27.18563138       34.02    0.0002
CR                       1        0.00001035        0.00    0.9972


                                 T FOR HO:    PR > |T|   STD ERROR OF
PARAMETER          ESTIMATE    PARAMETER=0                  ESTIMATE

INTERCEPT       43.31052314          40.10      0.0001     1.07994356
CU               0.08965208           0.13      0.8981     0.68037094
AT              -0.29139216          -5.83      0.0002     0.04995695
CR               0.0000840            0.00      0.9972     0.02335293
--------------------------------------------------------------------
```

Figure 23 Computer output (SAS) of general linear model procedure applied to LLDPE crystallinity data.

indicate that the developed model is adequate. The developed model is

$$CY = 43.31 - 0.29\ AT + 0.08\ CU + 0.00008\ CR$$

Residual Analysis

The results of residual and normal probability plots are shown in Figure 24. The plot of residuals does not indicate any serious model inadequacies. The scatter does not have any trend or curvature or inequality of variance. The residuals are also plotted on normal probability paper. Since the residuals fall approximately along a straight line, it is concluded that there is no severe departure from normality. These plots do not indicate any serious model inadequacies.

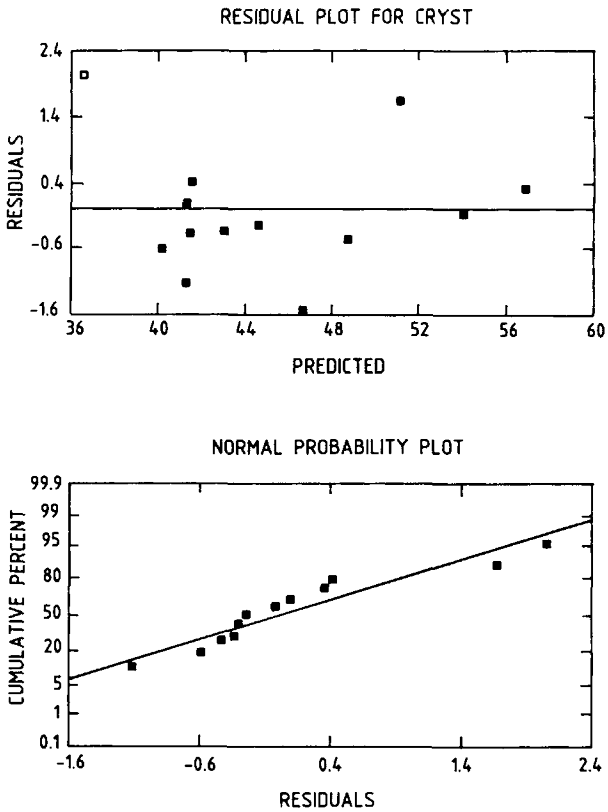


Figure 24 Residual and normal probability plot of LLDPE crystallinity (cryst) model.

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